

Numerical Linear Algebra

Projectors

1. Projectors

A *projector* is a square matrix P that satisfies

$$P^2 = P.$$

2. Complementary Projectors

If P is a projector, $I - P$ is also a projector,

$$(I - P)^2 = I - P.$$

The matrix $I - P$ is called the *complementary projector* to P .

Properties:

$$\text{range}(I - P) = \text{null}(P).$$

$$\text{null}(I - P) = \text{range}(P).$$

$$\text{range}(P) \cap \text{null}(P) = \{0\}.$$

3. Orthogonal Projectors

Theorem 1. A projector P is orthogonal if and only if $P = P^T$.

4. Projection with an Orthogonal Basis

$$P = QQ^T$$

where the columns of Q are orthonormal.

5. Projection with an Arbitrary Basis

$A \in \mathbb{R}^{m \times n}$ whose j th column is a_j . $\{a_1, \dots, a_n\}$ are linearly independent vectors.

$$P = A(A^T A)^{-1} A^T.$$