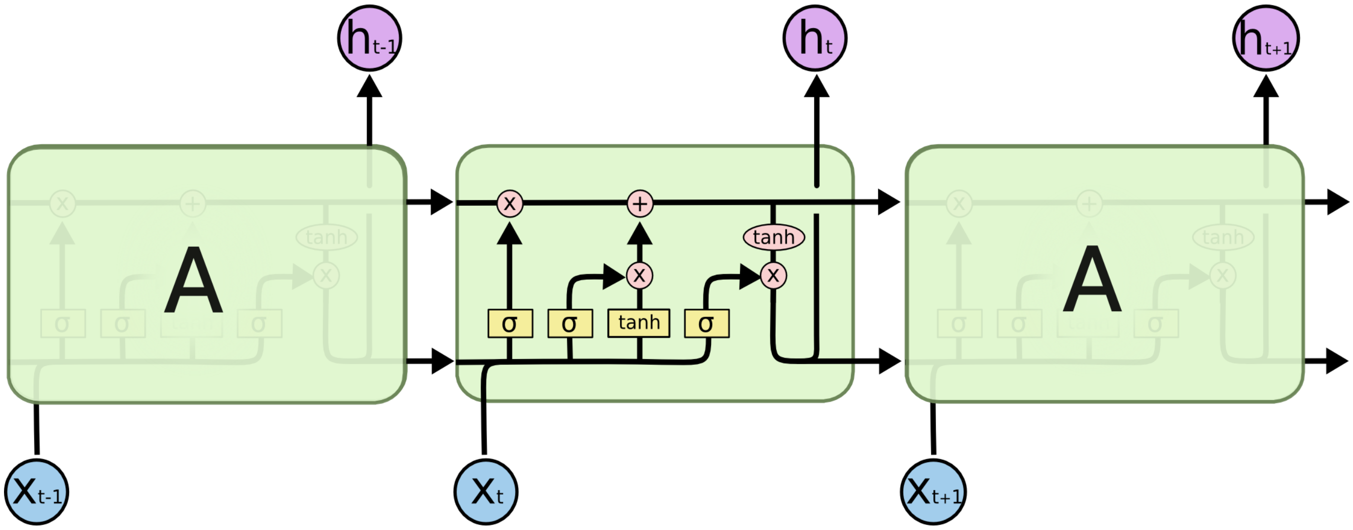
Machine Learning

LSTM Networks



1. Basics

sigmoid function:

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

$$\sigma'(x) = \sigma(x) \cdot [1 - \sigma(x)]$$

hyperbolic function:

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\tanh'(x) = 1 - \tanh^2(x)$$

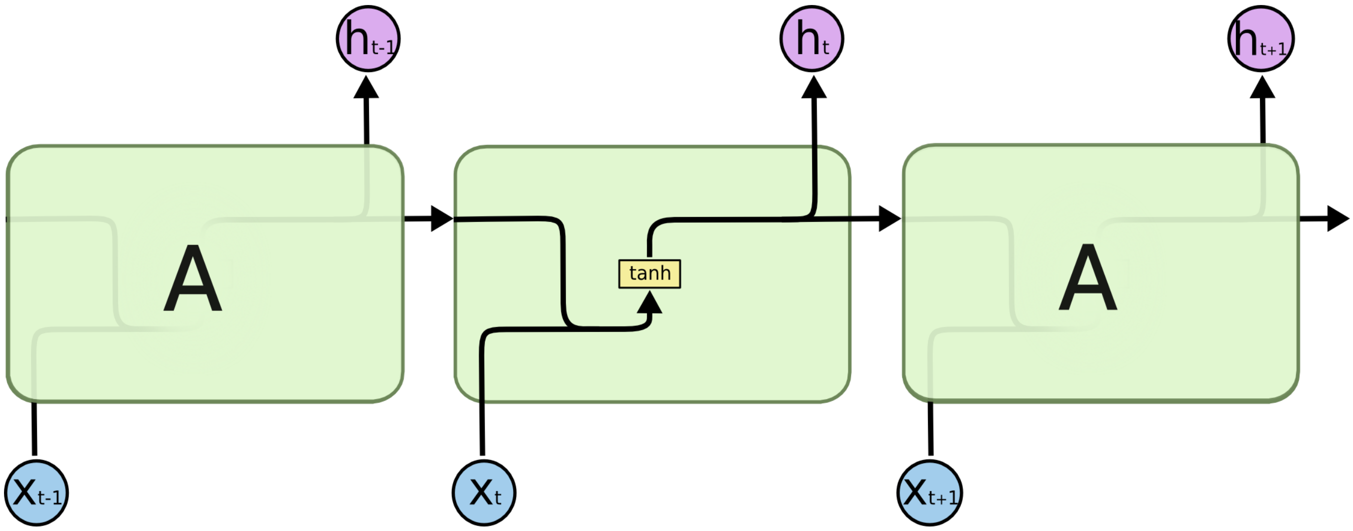
softmax function:

$$\mathbf{y} = \text{softmax}(\mathbf{x})$$

$$y_i = \frac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}$$

$$\frac{\partial y_i}{\partial x_j} = \begin{cases} -y_i \cdot y_j, & i \neq j \\ y_i \cdot (1 - y_i), & i = j \end{cases}$$

standard RNN:



2. Model

input:

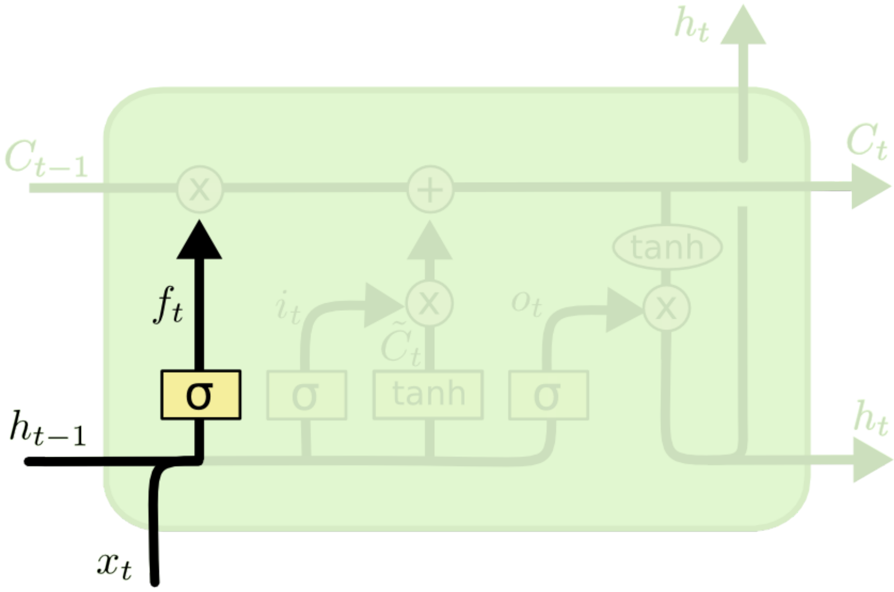
$$\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_T) \quad \mathbf{x}_t \in \mathbb{R}^n$$

initialize hidden state:

$$\mathbf{h}_0 \in \mathbb{R}^k$$

forget gate:

$$\mathbf{f}_t = \sigma(\mathbf{W}_{xf}\mathbf{x}_t + \mathbf{W}_{hf}\mathbf{h}_{t-1} + \mathbf{b}_f)$$

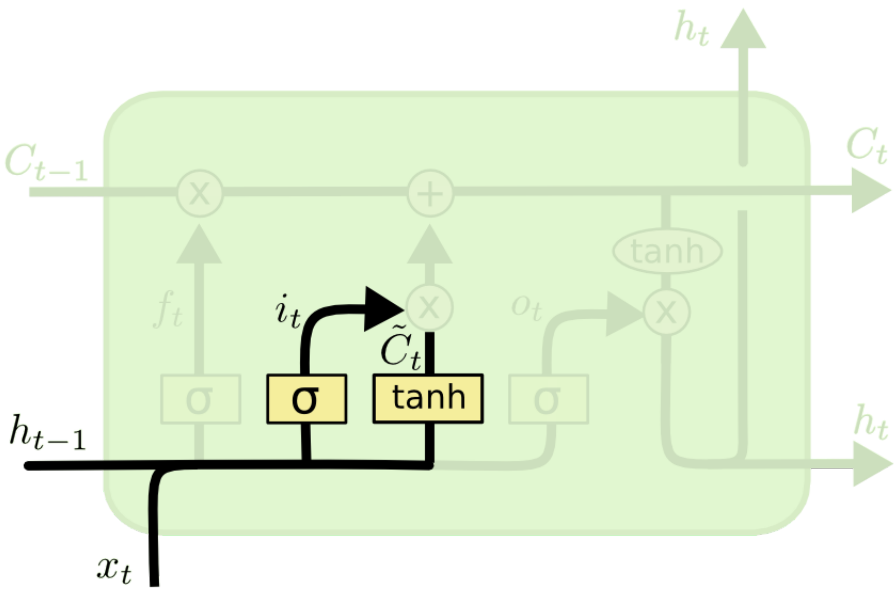


input gate:

$$\mathbf{i}_t = \sigma(\mathbf{W}_{xi}\mathbf{x}_t + \mathbf{W}_{hi}\mathbf{h}_{t-1} + \mathbf{b}_i)$$

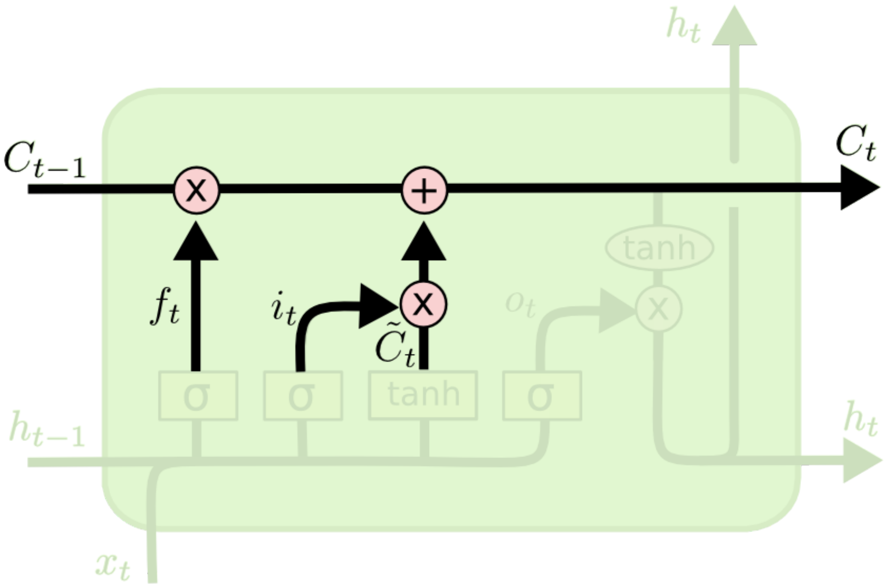
cell update transformation:

$$\tilde{\mathbf{c}}_t = \tanh(\mathbf{W}_{xc}\mathbf{x}_t + \mathbf{W}_{hc}\mathbf{h}_{t-1} + \mathbf{b}_c)$$



cell state update:

$$\mathbf{c}_t = \mathbf{f}_t \odot \mathbf{c}_{t-1} + \mathbf{i}_t \odot \tilde{\mathbf{c}}_t$$

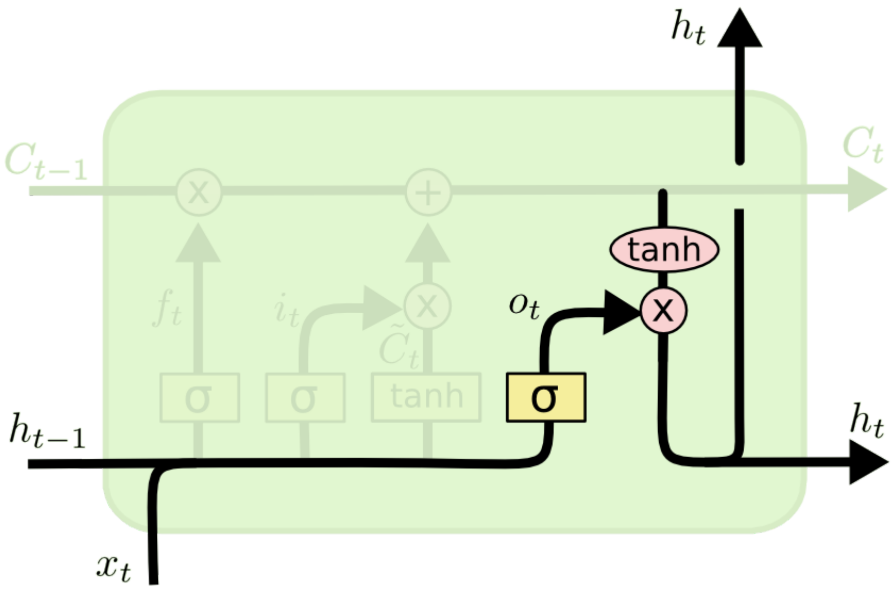


output gate:

$$\mathbf{o}_t = \sigma(\mathbf{W}_{xo}\mathbf{x}_t + \mathbf{W}_{ho}\mathbf{h}_{t-1} + \mathbf{b}_o)$$

hidden update:

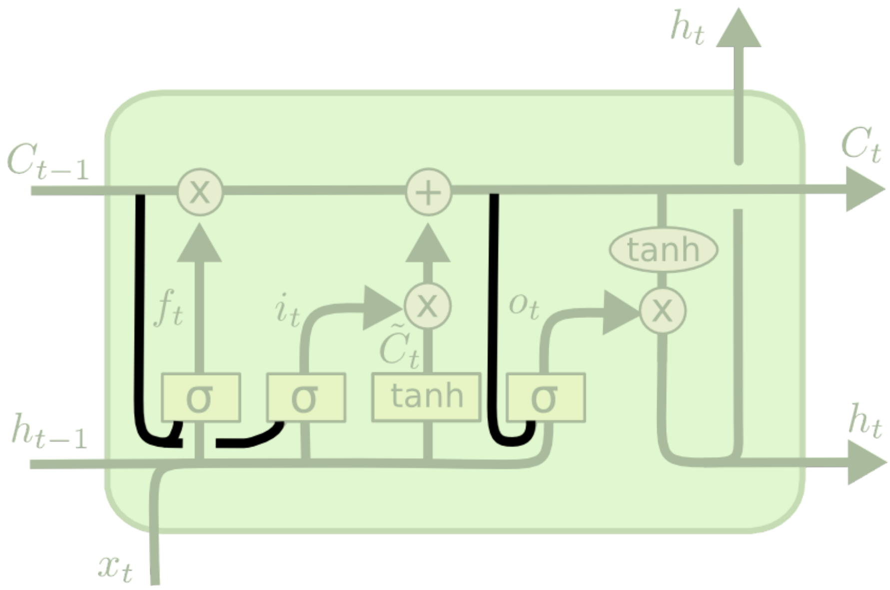
$$\mathbf{h}_t = \mathbf{o}_t \odot \tanh(\mathbf{c}_t)$$



3. Variants on LSTM

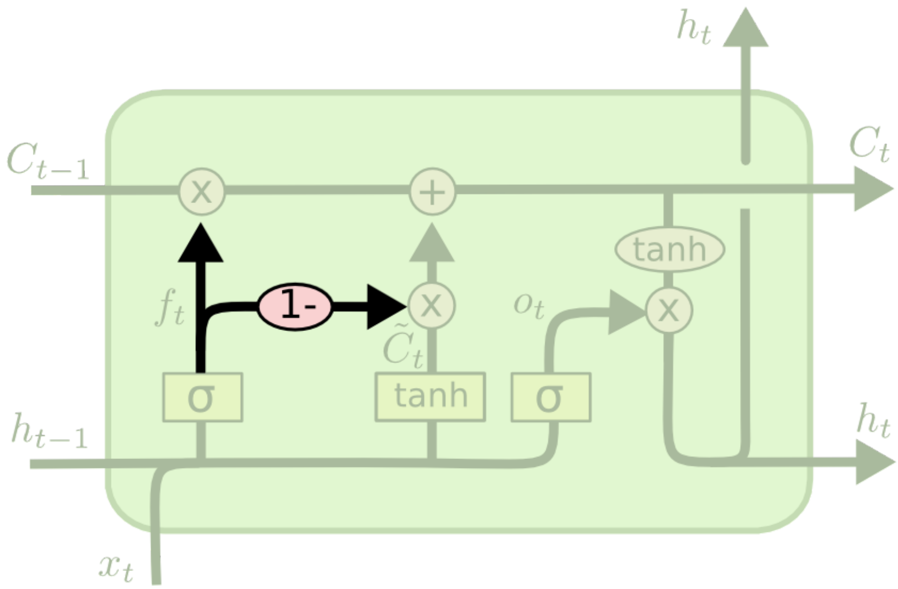
extending LSTM with peephole connections:

$$\begin{aligned}
\mathbf{i}_t &= \sigma(\mathbf{W}_{xi}\mathbf{x}_t + \mathbf{W}_{hi}\mathbf{h}_{t-1} + \mathbf{W}_{ci}\mathbf{c}_{t-1} + \mathbf{b}_i) \\
\mathbf{f}_t &= \sigma(\mathbf{W}_{xf}\mathbf{x}_t + \mathbf{W}_{hf}\mathbf{h}_{t-1} + \mathbf{W}_{cf}\mathbf{c}_{t-1} + \mathbf{b}_f) \\
\mathbf{o}_t &= \sigma(\mathbf{W}_{xo}\mathbf{x}_t + \mathbf{W}_{ho}\mathbf{h}_{t-1} + \mathbf{W}_{c0}\mathbf{c}_t + \mathbf{b}_o)
\end{aligned}$$



use coupled forget and input gates:

$$\mathbf{c}_t = \mathbf{f}_t \odot \mathbf{c}_{t-1} + (1 - \mathbf{f}_t) \odot \tilde{\mathbf{c}}_t$$



4. Gated Recurrent Unit(GRU)

update gate:

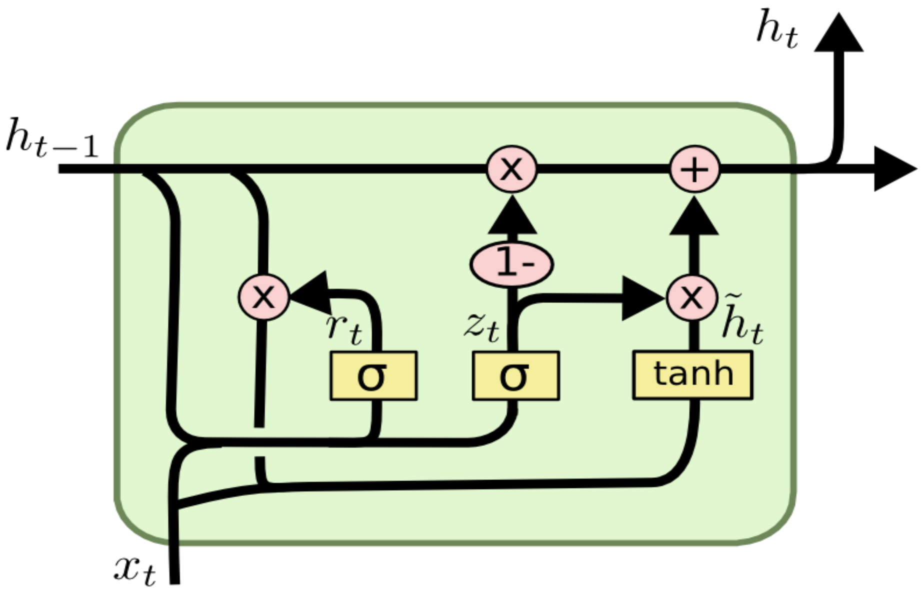
$$\mathbf{z}_t = \sigma(\mathbf{W}_{xz}\mathbf{x}_t + \mathbf{W}_{hz}\mathbf{h}_{t-1} + \mathbf{b}_z)$$

reset gate:

$$\mathbf{r}_t = \sigma(\mathbf{W}_{xr}\mathbf{x}_t + \mathbf{W}_{hr}\mathbf{h}_{t-1} + \mathbf{b}_r)$$

hidden update:

$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{W}_{xh}\mathbf{x}_t + \mathbf{W}_{hh}(\mathbf{r}_t \odot \mathbf{h}_{t-1}) + \mathbf{b}_h)$$
$$\mathbf{h}_t = (1 - \mathbf{z}_t) \odot \tilde{\mathbf{h}}_t + \mathbf{z}_t \odot \mathbf{h}_{t-1}$$



Reference

1. **Understanding LSTM Networks:** http://colah.github.io/posts/2015-08-Understanding-LSTMs/